Numerical modeling of suffusion as an interfacial erosion process

Frédéric Golay *, — Stéphane Bonelli***

** Unité de recherche Ouvrages Hydrauliques et Hydrologie, Cemagref*
3275 Route de Cézanne CS 40061
13182 Aix en Provence Cedex 5, France
stephane.bonelli@aix.cemagref.fr

*** Institut de Mathématique de Toulon, Université du Sud Toulon-Var*
Av. G. Pompidou BP 56
83162 La Valette du Var, France
frederic.golay@univ-tln.fr

ABSTRACT. *Suffusion is an internal erosion process which occurs when finer soil particles are detached by seepage flow and start moving within the pore space of larger particles. Suffusion can be viewed as clay/water interface erosion. The aim of this study is to use a numerical model for simulating surface erosion occurring at a fluid/soil interface subject to a flow process at pore scale. Balance equations with jump relations are used. A penalization procedure including a fictitious domain method is used to compute the Stokes flow around obstacles, in order to avoid body-fitted unstructured meshes and instead use fast and efficient finite volume approximations on Cartesian meshes. The evolution of the water/soil interface is described by using a Level Set function. The ability of the model to predict suffusion is confirmed by several numerical simulations.*

RÉSUMÉ. *La suffusion est un processus par lequel les particules les plus fines du sol se détachent de la matrice solide et sont transportées par un écoulement interstitiel à travers les pores du milieu. Le but de cette étude est de présenter un modèle numérique pour simuler l'érosion apparaissant à l'interface eau/sol à l'échelle des pores. Une procédure de pénalisation ou « domaine fictif » est utilisée pour simuler l'écoulement de Stokes autours d'obstacles, afin de s'affranchir des contraintes de maillage, grâce à un efficace solveur volume fini sur grille cartésienne. L'évolution de l'interface eau/sol est décrite par des fonctions Level Set. L'aptitude du modèle à prédire les phénomènes de suffusion est confirmée par quelques exemples.*

KEYWORDS: *Suffusion, internal erosion, finite volume, level set, interface, fictitious domain.*

MOTS-CLÉS : *Suffusion, érosion interne, volumes finis, level set, interface, domaines fictifs.*

European Journal of Environmental and Civil Engineering, Volume X – n° x/2010, pages 1 à X

1. Introduction

Suffusion occurs when finer soil particles are detached by seepage flow and start moving within the pore space of larger particles. This is an internal erosion process, that can be observed in non-uniform or broadly-graded soils. Internal erosion is a phenomenon that is not fully understood (Fell and Fry, 2007; Gutiérrez *et al.*, 2008). Understanding and modeling the removal of finer soil particles caused by flows of water through the pore domain is of major importance in geomechanics.

The initiation and development of internal erosion processes should be analyzed using a continuum approach at the macroscopic scale. However, this approach should also be considered at the microscopic scale when dealing with internal erosion. Both approaches must be consistent with each other. At pore scale, the suffusion process can be viewed as an interfacial erosion process. In this paper, the model of soil erosion developed by (Golay *et al.*, 2010) is applied to suffusion.

Several models of erosion of a sand layer have been proposed in the framework of the continuum theory of mixtures (Papamichos *et al.*, 2001; Vardoulakis *et al.*, 1996; Vardoulakis *et al.*, 2001; Chauchat *et al.*, 2010). In these studies, the erosion process is assumed to involve a smooth transition from solid-like to fluid-like behaviour. These models are relevant for permeable soils such as granular media. However, most cohesive soils like clay exhibit very low permeability and internal flows generated external flows are likely to be neglected in these soils.

The philosophy underlying the present approach differs from previous theories of surface erosion in that our description deals with singular (or discontinuous) fluid/soil interfaces (Bonelli and Brivois, 2008; Brivois *et al.*, 2007; Golay *et al.*, 2010; Lachouette *et al.*, 2008), rather than with smooth fluid/soil interfaces. The aim here is to model at micro scale the erosion of a cohesive fine soil generated by a pore laminar flow of water, tangential to the fine soil/water interface.

The paper is organized as follows: Section 2 summarizes the equations governing the Stokes fluid flow and the evolution of the fluid/soil interface at pore scale. Section 3 introduces, the mathematical model obtained by periodic homogenization, and a penalized model obtained with the fictitious domains method, leading to a unified formulation for the equations in the whole domain (fluid and soil). Finally, Section 4 presents several illustrative examples while section 5 presents the conclusions.

2. Governing equations

2.1. *Pore-scale flow with surface erosion of the clay matrix*

The porous medium is considered as an assembly of Representative Elementary Volumes (REV) of the microstructure (Fig. 1). The REV occupies a total volume Ω_{REV} of typical length l_{REV}. This REV is partitioned into three disjoint regions: a connected pore domain Ω_{pore}, a clay matrix domain Ω_{clay} and a sand grain domain Ω_{sand}. The pore domain is saturated by a viscous incompressible fluid of density ρ_f and viscosity μ_f.

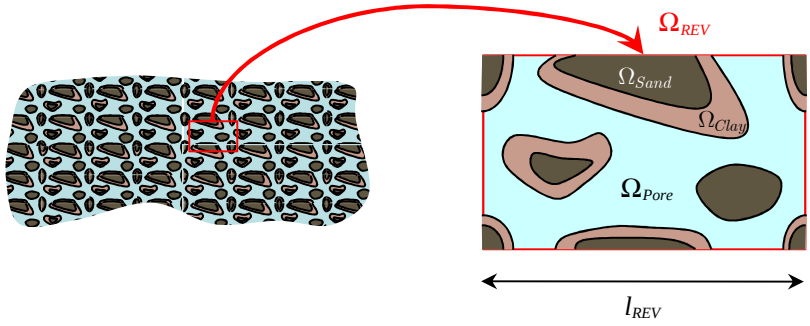

Figure 1. *Schematic of Representative Elementary Volume*

For the sake of simplicity, physico-chemical effect (like dissolution and deposition) are not considered. The clay matrix is a rigid and homogeneous erodible material, of density ρ_{clay}. The sand grains are rigid, homogeneous and non erodible. Rigid clay and sand have zero velocity: $\mathbf{u} = 0$ in Ω_{Clay} and Ω_{Sand}.

The Reynolds number is small and inertial effects are neglected. The Stokes equations corresponding to this steady and slow viscous flow in the pore domain are

$$\nabla \cdot \mathbf{u} = 0 \text{ in } \Omega_{Pore} \quad [1]$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla p \text{ in } \Omega_{Pore} \quad [2]$$

$$\boldsymbol{\tau} = 2\mu_f \mathbf{D}(\mathbf{u}) \text{ in } \Omega_{Pore} \quad [3]$$

where ∇ is the differential operator with respect to the microscopic space variable $\mathbf{x}$, p is the fluid pressure, $\mathbf{u}$ is the fluid velocity vector, and $\boldsymbol{\tau}$ is the viscous deviatoric stress tensor. The deviatoric strain rate tensor is

$$\mathbf{D}(\mathbf{u}) = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla^T \mathbf{u} - \frac{1}{3}(\nabla \cdot \mathbf{u})\,\mathbf{I}\right) \quad [4]$$

2.2. *Surface erosion of the clay matrix*

We take Γ to denote any fluid/solid interface, and $\mathbf{n}$ to denote the unit vector normal to Γ oriented from the fluid toward the soil (Fig. 2). In our description, Γ is a purely geometric separating surface with no thickness. We take $[\![a]\!]$ to denote the jump of any physical variable a across Γ.

As usual, no slip condition is assumed on the fluid/solid interface Γ:

$$\mathbf{u}_T = 0 \text{ on } \Gamma \quad [5]$$

where $\mathbf{u}_T = [\mathbf{I} - \mathbf{n} \otimes \mathbf{n}] \cdot \mathbf{u}$ is the tangential flow velocity on Γ.

The jump equations on the sand/water interface give the continuity of both the normal velocity and the stress vector

$$\mathbf{u} \cdot \mathbf{n} = 0 \text{ on } \Gamma_{Sand} \cap \Gamma \quad [6]$$

$$[\![\boldsymbol{\tau} \cdot \mathbf{n} - p\mathbf{n}]\!] = 0 \text{ on } \Gamma_{Sand} \cap \Gamma \quad [7]$$

We assume that erosion occurs only at the interface $\Gamma_{Clay} \cap \Gamma$ of the clay matrix and the pore domain, where $\Gamma_{Clay} = \partial\Omega_{Clay}$ and $\Gamma = \partial\Omega_{Pore}$. The erosion process is therefore external to the clay matrix, and does not affect its porosity or density.

Due to the erosion process, a mass flux crosses $\Gamma_{Clay} \cap \Gamma$. As a result, $\Gamma_{Clay} \cap \Gamma$ is a moving interface, but not a material interface: Γ is not defined by the same particles at different times. We take c_Γ to denote the normal celerity of Γ.

The total flux of eroded material $\dot{m}$ crossing $\Gamma_{Clay} \cap \Gamma$ is defined as follows:

$$\dot{m} = \rho(c_\Gamma - \mathbf{u} \cdot \mathbf{n}) \text{ on } \Gamma_{Clay} \cap \Gamma \quad [8]$$

ρ denotes the total density of the fluid in Ω_{Pore} and the density of the soil in Ω_{Clay}. The jump equations on the clay/water interface yields the continuity of the mass flux of eroded material and the relationship between the stress vector and the velocity:

$$[\![\dot{m}]\!] = 0 \text{ on } \Gamma_{Clay} \cap \Gamma \quad [9]$$

$$[\![\boldsymbol{\tau} \cdot \mathbf{n} - p\mathbf{n}]\!] = -\dot{m}[\![\mathbf{u}]\!] \text{ on } \Gamma_{Clay} \cap \Gamma \quad [10]$$

Shear induced surface erosion is described with the following constitutive erosion law:

$$\dot{m} = \begin{cases} k_{er}(\tau - \tau_c) \text{ if } \tau > \tau_c \\ 0 \text{ otherwise} \end{cases} \text{ on } \Gamma \quad [11]$$

$$\tau = \left\| [\mathbf{I} - \mathbf{n} \otimes \mathbf{n}] \cdot \boldsymbol{\tau} \cdot \mathbf{n} \right\| \text{ on } \Gamma \quad [12]$$

where τ is the tangential shear stress at the interface. Parameter τ_c is the threshold shear stress and parameter k_{er} is the coefficient of surface erosion.

The no-slip assumption Eq. [5] and the momentum jump Eq. [10] imply the continuity of the tangential shear stress across fluid/solid interface Γ ($[\![\tau]\!] = 0$).

The continuity of the mass flux of eroded material Eq. [9] gives the relationship between this mass flux and the interface normal celerity:

$$\dot{m} = \rho_{Clay} c_\Gamma \text{ on } \Gamma_{Clay} \cap \Gamma \qquad [13]$$

Here, we assume a slow erosion process: $c_\Gamma \ll \|\mathbf{u}\|$. This key assumption has several consequences at the first order approximation:

- the continuity of the mass flux of eroded material Eq. [9] implies the continuity of normal velocities, i.e. $\mathbf{u} \cdot \mathbf{n} = 0$ on Γ;

- the momentum jump Eq. [10] shows that normal stresses are continuous across Γ, i.e. $[\![\mathbf{n} \cdot \boldsymbol{\tau} \cdot \mathbf{n} - p]\!] = 0$ on Γ;

- the quantity of particles present in the fluid can be assumed to be small enough not to significantly affect the properties of the carrier fluid; this is the dilute suspension flow assumption.

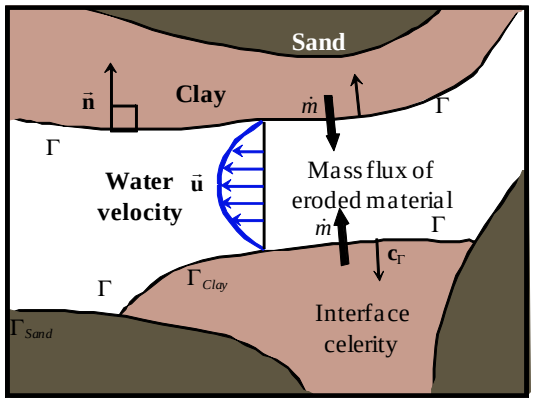

Figure 2. *Diagram of the suffusion process viewed as the surface erosion of the clay matrix inside a pore*

2.3. *Orders of magnitude*

Let us now check what the important physical parameters of the problem are. We take L to denote the macroscopic characteristic length scale, and Δp^0 to denote the characteristic variation of pressure p over L. For example, $L \gg \ell_{REV}$ may be the

sample length in a laboratory test. These quantities give the order of magnitude of the macroscopic pressure gradient, which defines the loading.

We take ℓ_f to denote the characteristic length scale of the sheared fluid at the pore scale, which is a given parameter of the system. At the first order, ℓ_f can be considered to have the same order of magnitude as the sand grain diameter. For the sake of clarity, we introduce the characteristic tangential stress on the interface τ_Γ, the characteristic microscopic fluid velocity V_f and the characteristic celerity of the fluid/solid interface during erosion V_{er}.

Orders of magnitude are given by Eq. [2] for the shear stress, by Eq. [3] for the fluid velocity, and by Eqs. [11] and [13] for the erosion rate, as follows:

$$\tau_\Gamma \sim \frac{\ell_f \Delta p^0}{L},\ V_f \sim \frac{\ell_f \tau_\Gamma}{\mu_f},\ V_{er} \sim \frac{k_{er} \tau_\Gamma}{\rho_{Clay}} \qquad [14]$$

The Reynolds number and the erosion number of the flow are defined as follows:

$$R_e = \frac{\rho_f V_f^2}{\tau_\Gamma},\ \mathcal{E}_r = \frac{\rho_f V_f k_{er}}{\rho_{Clay}} \qquad [15]$$

The assumptions of a laminar flow ($R_e \ll 1$) and a slow erosion process ($\mathcal{E}_r \ll 1$) are written as:

$$\frac{\Delta p^0}{L} << \frac{\mu_f}{\rho_f \ell_f^2} \min\left(\frac{\mu_f}{\ell_f}, \frac{\rho_{Clay}}{k_{er}}\right) \qquad [16]$$

The erosion number was introduced by Bonelli and Brivois (2008), and (Golay *et al.*, 2011), and represents the ratio between the fluid velocity and the erosion velocity ($\mathcal{E}_r = R_e V_{er} / V_f$). From this finding, we infer the physical meaning of the coefficient of erosion k_{er} (s/m). This coefficient, which drives the erosion process and depends on the eroded material considered, may be viewed as a ratio between the surface clay viscosity μ_{Clay} and a microscopic length scale characteristic of the erosion process ℓ_{er}: $k_{er} = \rho_{Clay} \ell_{er} / \mu_{Clay}$. In a sense, μ_{Clay} characterize the time scale of tear resistance of soil particles.

The dilute flow assumption corresponds to $V_{er} \ll V_f$, written as:

$$\frac{k_{er}}{\rho_{Clay}} \ll \frac{\ell_f}{\mu_f} \qquad [17]$$

The physical meaning of ℓ_{er} remains to be defined. However, this length scale cannot be greater than the pore length scale : $\ell_{er} \leq \ell_f$. We assume therefore that ℓ_{er} and ℓ_f are of the same order of magnitude : $\ell_{er} = \ell_f$. With this assumption, the dilute flow assumption reads $\mu_f \ll \mu_{Clay}$. Orders of magnitude are as follows: $\mu_f = 10^{-3}$ Pa.s and $\mu_{Clay} > 1$ Pa.s for bulk clay viscosity. This is consistent with our assumption of slow erosion process and dilute flow.

The characteristic timescale linked to the interfacial erosion process is $t_{er} = \ell_f / V_{er}$ (Bonelli and Brivois, 2008). In the present situation dealing with suffusion driven by a pressure gradient, this timescale turns out to be:

$$t_{er} = \frac{\rho_{Clay} L}{k_{er} \Delta p^0} \quad [18]$$

3. The numerical model

3.1. *Homogenization and governing equations at the first order*

We now assume that the domain is a periodic medium. A complete set of equations similar to equations [1]-[13] has already been used to study different systems by using periodic homogenization (Bouddour *et al.*, 1996). It is assumed that the REV length l_{REV} and the characteristic microscopic length ℓ_f are small compared to the macroscopic characteristic length scale L: $\ell_f < l_{REV} \ll L$. The periodic homogenization procedure leads to the following well-known results:

- the zero-order pore water pressure p^0 is independent of local coordinate x:

$$\nabla p^0 = 0 \text{ in } \Omega_{Pore} \quad [19]$$

- the governing equation for the higher order pore water pressure p^1 and for the zero-order microscopic velocity u^0 is

$$\nabla \cdot \mathbf{u}^0 = 0 \text{ in } \Omega_{Pore} \quad [20]$$

$$\mu_f \Delta \mathbf{u}^0 = \nabla p^1 + \frac{\partial p^0}{\partial \mathbf{X}} \text{ in } \Omega_{Pore} \quad [21]$$

$$\mathbf{u}^0 = 0 \text{ on } \Gamma \quad [22]$$

$$\mathbf{u}^0 \text{ and } p^1 \text{ are } \Omega_{\mathrm{Re}v}\text{-periodic} \quad [23]$$

$$c_\Gamma = \begin{cases} \dfrac{k_{er}}{\rho_{Clay}} (\tau^0 - \tau_c) \text{ if } \tau > \tau_c \\ 0 \text{ otherwise} \end{cases} \text{ on } \Gamma_{Clay} \cap \Gamma \quad [24]$$

$$c_\Gamma = 0 \text{ on } \Gamma_{Sand} \cap \Gamma \quad [25]$$

$$\tau^0 = 2\mu_f \left\| [\mathbf{I} - \mathbf{n} \otimes \mathbf{n}] \cdot \mathbf{D}(\mathbf{u}^0) \cdot \mathbf{n} \right\| \text{ on } \Gamma \quad [26]$$

where $\partial p^0 / \partial \mathbf{X}$ denotes the macroscopic pressure gradient which is constant on the REV. This vector is the loading of the system and the order of magnitude is $\| \partial p^0 / \partial \mathbf{X} \| = \Delta p^0 / L$.

3.2. *The fictitious domain method*

The aim here is to simulate a fluid flow with a moving boundary Γ . A classical method consists in building a mesh of the fluid part with cell edges belonging to Γ . It is necessary to regularly adapt the mesh or remesh as the boundary varies. This numerical process can become prohibitive, especially when considering a three-dimensional problem. That is why a spatial discretization that is not fitted to the interface is used. This method is called the "fictitious domain method" or "penalized method". The main idea of this approach consists in solving the fluid flow problem in a bigger fixed domain instead of the moving fluid domain. This method is popular because it allows the use of fairly structured meshes. The fictitious domain method is now widely used (in particular for fluid-porous-solid flows) and has been mathematically justified (see e.g. Angot *et al.*, 1999; Angot, 1999; Bost *et al.*, 2010; Diaz-Goano *et al.*, 2003; Khadra *et al.*, 2000; Ramiere *et al.*, 2007).

Let $\Omega_{REV} = \Omega_{Pore} \bigcup (\underset{i}{\cup} \Omega_i)$ be a fluid and multi-solids domain with boundary $\partial\Omega$. The penalized method allows describing the behavior of the two sub-domains by using equations valid for the whole domain. This is done by introducing a new term in the momentum balance equation, this time defined in Ω_{REV}

$$\mu_f \Delta \mathbf{u}^0 = \nabla p^1 + \frac{\partial p^0}{\partial \mathbf{X}} + \frac{\mu_f}{K_s} (\underset{i}{\Sigma} H^i) \mathbf{u}^0 \text{ in } \Omega_{REV} \qquad [27]$$

Penalization coefficient K_s is a small number that has the dimension of a geometric permeability and H^i is the characteristic function of the soil domain *i* (which is unity within Ω_i and zero elsewhere). In practice we use often $K_s = 10^{-7}$ or $K_s = 10^{-9}$.

As the last term of the right-hand side of Eq. [27] is zero in the fluid domain, the equation reduces to the Stokes equation. In the soil domain, penalization leads to very low values of $\mathbf{u}^0$ and the left-hand side of Eq. [27] is negligible. This time the equation reduces to a Darcy like equation.

3.3. *Determination of the interface by the Level Set Method*

The Level Set method is an appropriate formulation for considering sharp interfaces. It consists in introducing a function ψ that is negative in the fluid domain Ω_{Pore} and positive in the solid domain. In the case of several solid domains, we introduce several functions ψ^i that are positive in the solid domains Ω_i and negative elsewhere.

Interface Γ is represented by the zero level set of ψ^i. The characteristic function H^i of the soil domain i is therefore defined as follows:

$$H^i = \begin{cases} 1 \text{ if } \psi^i > 0, \\ 0 \text{ otherwise} \end{cases} \quad [28]$$

A possible and relevant approach is to choose this function as the signed distance to the interface. The motion of the interface is thus determined by the evolution of ψ^i. Keeping track of the whole function ψ^i while only its zero level set is relevant may appear inefficient, but it in fact simplifies both the mathematical formulation and numerical implementation.

This approach can be considered as a front capturing method as no explicit information on the interface is required during the computational process. The interface is recovered at the end of computation by locating the zero level. Cartesian meshes are used because capturing methods do not require body-fitted grids instead of front tracking methods. The accuracy of the definition of the interface strongly depends on the numerical scheme applied to the transport equation.

This Level Set formulation was introduced by Osher and Sethian (1981) and is widely used for multi-fluid flows (e.g. Chang *et al.*, 1996; Olson and Kreiss, 2005; Sussman *et al.*, 1994), with a fictitious domain approach (e.g. Chantalat *et al.*, 2009) in both two and three-dimensional cases (e.g. Prodanovic and Bryant, 2006).

Assuming that the zero level set of ψ^i and the interface coincide at $t = 0$, then they must coincide at all times provided that ψ^i satisfies $\partial \psi^i / \partial t + \mathbf{c}_\Gamma \cdot \nabla \psi^i = 0$ at the interface. This condition is automatically satisfied assuming that ψ^i is then driven by a transport equation:

$$\frac{\partial \psi^i}{\partial t} + \mathbf{c} \cdot \nabla \psi^i = 0 \quad [29]$$

where $\mathbf{c}$ is an extension on the whole domain of the interface celerity $\mathbf{c}_\Gamma$ defined on the interface (Golay *et al.*, 2011). The normal to the interface is given by the gradient of the level set function:

$$\mathbf{n}^i = \frac{\nabla \psi^i}{\left\| \nabla \psi^i \right\|} \quad [30]$$

As the erosion process is much slower than the main fluid flow, the two phenomena are split. For the Stokes solver, a finite volume formulation is developed on a Cartesian staggered grid (Harlow and Welch, 1965): the physical domain is given by the pressure mesh, while the velocity is given at the centre of each face of the cell. We use the Augmented Lagrangian method in order to deal with the constraint of divergence-free velocity (see e.g. Fortin and Glowinski, 1983; Vincent *et al.*, 2004; Galusinski and Vigneaux, 2008).

For the erosion process, spatial derivatives are computed with a fifth-order WENO (weighted essentially non-oscillatory) scheme (Jiang & Peng, 2000), while time derivatives are computed with a fourth-order Runge-Kutta (RK4) scheme.

The tangential stress at the interface must be estimated with accuracy. A detailed description of our numerical model is given in Golay *et al.* (2011).

As a finite volume method is used, fluxes crossing the boundary of each cell (faces) are computed, and a Cartesian grid ensures that the discretization of each periodic side is the same. The periodicity is than naturally and simply introduced, by declaring that the cell adjacent to one side of the REV is the neighbor of the opposite cell on the periodic side (see e.g. Guus et al., 1994).

4. Results

4.1. *Erosion of soil cylinders*

We considered a simple test case, namely a Stokes flow under a constant pressure gradient in a 2D periodic medium. The test simulated the erosion of fixed cylinders of soil in a channel (Figure 3). The soil cylinders had a radius of 3mm and a density of 2000 kg/m^3. Each REV had a length and height of 1cm respectively. The fluid inside had a density of 1000 kg/m^3 and a viscosity of 10^{-3} Pa.s. We considered an erosion process without threshold (τ_c =0) but with erosion coefficient k_{er} =10^{-3} s/m, which led to an erosion timescale of t_{er} = 2.10^5 s . A regular mesh of 100x100 and periodic conditions on the boundaries of the REV were implemented.

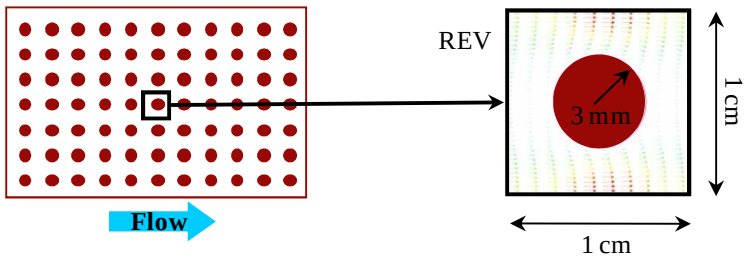

Figure 3. *Erosion of soils cylinders in a 2D periodic medium. The arrow shows the flow direction and magnitude.*

We imposed a macroscopic pressure gradient from left to right (Δp^0 =0.1 Pa). Figure 4 shows the evolution of the cylinder shape due to erosion. The line

represents the zero Level Set function, i.e. the shape of the eroded soil, and the vectors are the velocity field. It can be seen that, as expected for the Stokes flow, the shape and flow are symmetrical, and that the upper and lower parts of the cylinder are those most eroded.

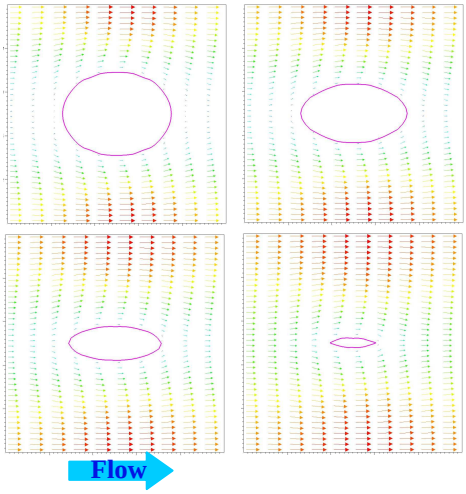

Figure 4. *Shape of the soil cylinder at* t / t_{er} *=0.13, 0.26, 0.36, 0.44. The arrow shows the flow direction and magnitude.*

4.2. *Erosion of four balls*

In order to check the feasibility of such computation in three-dimensions, we tested the erosion of four spheres in a cubic REV of length 1cm, subjected to a constant pressure gradient of 0.1Pa/cm. The test simulated the erosion of fixed spheres of soil in a channel (Figure 5). The soil spheres had a radius of 2mm and a density of 2000 kg/m^3. The fluid inside had a density of 1000 kg/m^3 and a viscosity of 10^{-3} Pa.s. We considered an erosion process without threshold (τ_c =0) but with erosion coefficient k_{er} =10^{-3} s/m, which leads to an erosion timescale of $t_{er} = 2.10^5$ s. A regular mesh of 40x40x40 and periodic conditions on the boundaries of the REV were implemented. This simulation was computed with only one Level Set function. The velocity field increased as the pressure gradient was imposed and the soil eroded.

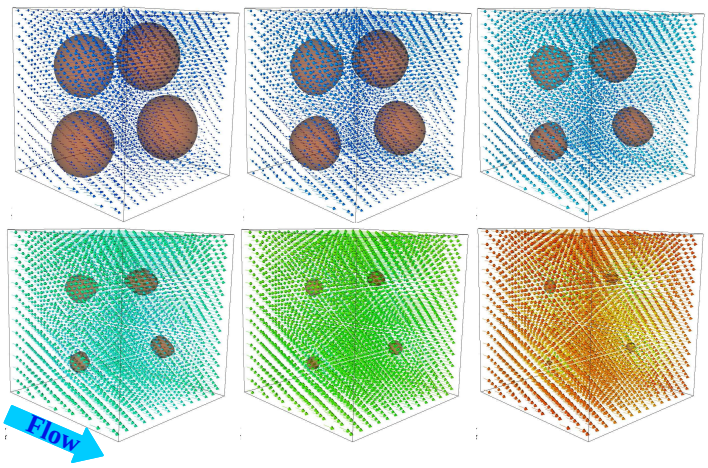

Figure 5. *Shape of the eroded balls at* t / t_{er} *= 0, 0.135, 0.25, 0.315, 0.355, 0.41. The arrow shows the flow direction and magnitude.*

4.3. *Suffusion*

In order to demonstrate that interfacial erosion at the micro-scale can simulate the suffusion process, we considered a representative elementary volume composed of thirty randomly placed sand grains surrounded by clay (Figure 6). Consequently, a porous medium (porosity 0.386) was obtained in which the sand grains were non erodible while the clay matrix was. The sand grains had a radius 300 µm while that of the particles of the clay layer was 640 µm with a density of 2000 kg/m^3. The REV was 1cm long and 1cm high. The fluid had a density of 1000 kg/m^3 and a viscosity of 10^{-3} Pa.s. We considered an erosion process with a threshold of 1Pa and an erosion coefficient of 10^{-5} s/m. The erosion timescale was $t_{er} = 1\times10^5$ s with a reference pressure drop Δp^0=20 Pa. A regular mesh of 150x150 was used with nd periodic conditions on the boundaries of the REV

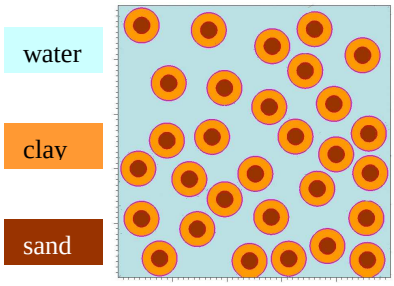

Figure 6. *Model of suffusion: representative elementary volume*

Let α denote the ratio of the current volume fraction of clay divided by the initial volume fraction of clay, i.e. if $\alpha = 1$ no clay is eroded, if $\alpha = 0$ all clay is eroded. The effect of varying the pressure gradient is shown in Figure 7. The erosion process increased as the macroscopic pressure gradient rose. For low values, the erosion process stops because the shear stress of the flow becomes lower than the microscopic threshold shear stress, so that some residual clay remains. Qualitatively, these results are in concordance with those obtained by (Sterpi, 2003). Figure 8 presents the evolution of the REV under a macroscopic pressure gradient $\Delta p^0 = 30$ Pa.

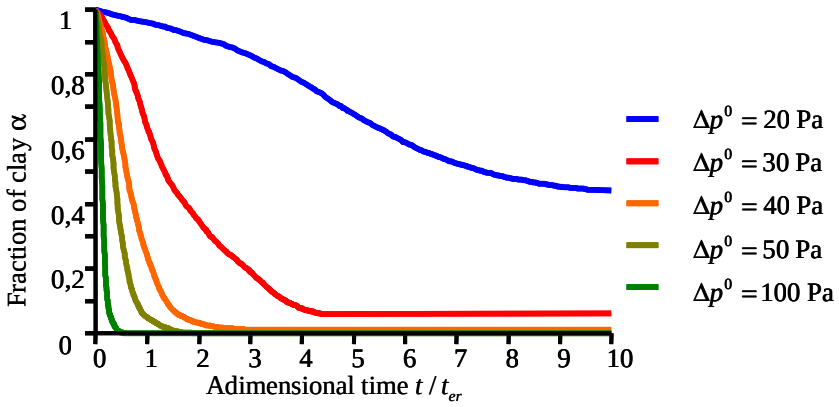

Figure 7. *Evolution of volume fraction of clay with time*

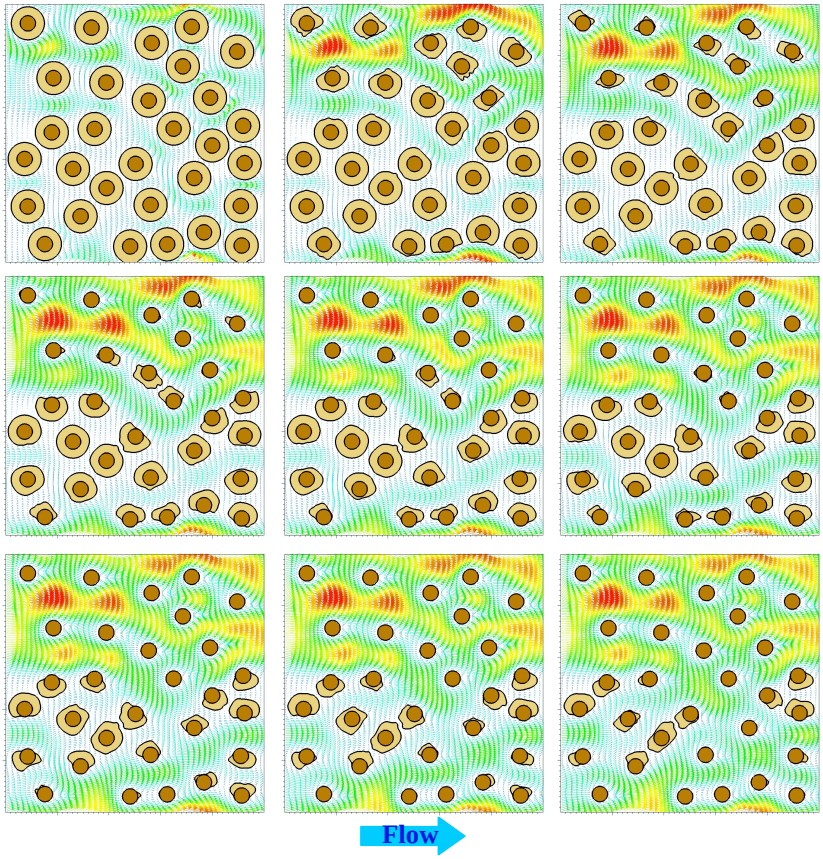

Figure 8. *Evolution of the REV at* t / t_{er} *= 0, 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4. The arrow shows the flow direction and magnitude.*

5. Conclusion

Soil erosion is the removal of material caused by the eroding power of a water flow and this is basically an interfacial process. The goal of this work was to better understand the suffusion erosion process, which is usually considered as a bulk process at the macroscopic scale. At the pore scale, it can be viewed as an interfacial erosion process.

We developed a numerical model for simulating surface erosion occurring at a fluid/soil interface subject to a flow process at pore scale, on a periodic representative elementary volume. Balance equations with jump relations were used. A penalization procedure allowed computing Stokes equations around obstacles, by using a fictitious domain method to avoid body-fitted unstructured meshes and use fast and efficient finite volume approximations on Cartesian meshes. The water/soil interface evolution was described with a Level Set function. The qualitative comparison between the results of the present modelling study and previously published experimental data supports the validity of our approach.

The present work is not intended to provide accurate numerical results for a suffusion related issue. Rather, attention is focused explicitly on the more limited goal of bridging the gap between the counter-intuitive bulk erosion model and our intuition that erosion is basically an interfacial process and on giving trends and orders of magnitude. The main result of this approach is as follows: the coefficient of surface erosion appears to be a relevant parameter for the suffusion bulk erosion law. Of course, other important phenomena should be considered in a comprehensive model of suffusion with a more realistic REV, like deposition and clogging, two-phase seepage flow and concentrated flow, dissolution and physico-chemical effects. For a better description of a realistic media, we intend to develop several numerical improvements such as implementing a parallel solver on unstructured grids.

Acknowledgments: This work was funded by the French National Research Agency (ANR) through the COSINUS program (CARPEINTER project no.ANR-08-COSI-002).

6. References

Angot P., Bruneau, C.-H., Fabrie, P., "A penalization method to take into account obstacles in incompressible viscous flows", *Numer. Math.,* Vol. 81, 497-520, 1999.

Angot P., "Analysis of Singular Perturbations on the Brinkman Problem for Fictitious Domain Models of Viscous Flows", *Math. Meth. Appl. Sci.,* 22, 1395-1412, 1999.

Bonelli S., Brivois O., "The scaling law in the hole erosion test with a constant pressure drop", *Int. J. Num. Meth. Eng.,* Vol. 32, 1573-1595, 2008.

Bost C., Cottet G.-H., Maitre E., “Convergence analysis of a penalization method for the three-dimensional motion of a rigid body in an incompressible viscous fluid”, *SIAM J. Numer. Anal*., 48 (4), 1313-1337, 2010.

Bouddour A., Auriault J. L., Mhamdi-Alaoui M., “Erosion and deposition of solid particles in porous media: Homogenization analysis of a formation damage”, *Transport Porous Med*, 25(2): 121-146, 1996.

Brivois, O., Bonelli, S., Borghi, R., "Soil erosion in the boundary layer flow along a slope: a theoretical study", *Eur. J. Mech. B-Fluid.*, Vol. 26, 707-719, 2007.

Chang Y.C., Hou T.Y., Merriman B., Osher S., "A Level Set formulation of eulerian interface capturing methods for incompressible fluid flows", *J. Comput. Phys.*, 124, 449-464, 1996.

Chantalat F., Bruneau C.H., Galusinski C., Iollo A., "Level-set, penalization and cartesian meshes: A paradigm for inverse problems and optimal design", *J. Comput. Phys.*, 228, 6291-6315, 2009.

Chauchat J., Médale M., "A three-dimensional numerical model for incompressible two-phase flow of a granular bed submitted to a laminar shearing flow", *Comput. Methods Appl. Mech. Engrg*., 199 (9-12), 439-449, 2010.

Diaz-Goano C., Minev P.D., Nandakumar K.," A fictitious domain/finite element method for particulate flows", *J. Comput. Phys.*, 192, 105-123, 2003.

Fell R., Fry J.-J., *Internal erosion of dams and their foundations*, Taylor & Francis, London, 2007.

Fortin M., Glowinski R., " Augmented lagrangian methods: applications to the numerical solution of boundary-value problems ", *studies in mathematics and its applications*, vol. 15, North-Holland, Amsterdam, 1983.

Galusinski C., Vigneaux P., "On stability condition for bifluid flows with surface tension: application to microfluidics", *J. Comput. Phys.*, 227, 6140-6164, 2008.

Guus S, Kees V., Kees K., "On the implementation of symetric and antisymetric periodic boundary conditions for incompressible flow", *Int. J. Num. Meth. Fluids*, 18, 1153-1165, 1994.

Gutiérrez F., Guerrero J., Lucha P., " A genetic classification of sinkholes illustrated from evaporate paleokarst exposures in Spain ", *Environ. Geol.*, 53 (5), 993-1006, 2008.

Golay F., Lachouette D., Bonelli S., Seppecher P., “Interfacial erosion: A three-dimensional numerical model”, *C.R. Mecanique*, 338 (6), 333-337, 2010.

Golay F., Lachouette D., Bonelli S., Seppecher P., "Numerical modelling of interfacial soil erosion with viscous incompressible flows", *Comput. Methods Appl. Mech. Engrg*, 200 (1:4), 383-391, 2011.

Harlow FH, Welsh JE. "Numerical calculation of time dependent viscous incompressible flow with free surface", *Phys Fluids*, 8 (12), 2182–2189, 1965.

Jiang G.S., Peng D., "Weighted ENO schemes for Hamilton-Jacobi equations", *J. Sci. Comput.*, Vol. 21(6), 2126-2143, 2000.

Khadra K., Angot P., Parneix S., Caltagirone,J.P., "Fictitious domain approach for numerical modelling of Navier-Stokes equations", *Int. J. Numer. Meth. Fluids*, 34, 651-684, 2000.

Lachouette, D., Golay, F., Bonelli, S., "One-dimensional modeling of piping flow erosion", *C. R. Mecanique*, Vol. 336, 731-736, 2008.

Olson E., Kreiss G., "A conservative level set method for two phase flow", *J. Comput. Phys.*, 210, 225-246, 2005.

Osher, S., Sethian, J."A., Fronts propagating with curvature-dependent speed: Algorithms for tracking material interface", *J. Comp. Phys.*, Vol. 39, 201-225, 1981.

Papamichos E, Vardoulakis I, Tronvoll J, Skjaerstein "A Volumetric sand production model and experiment", *Int. J. Num. Anal. Meth.*, 25, 789-808, 2001.

Prodanovic M., Bryant S., "A level set method for determining critical curvatures for drainage and imbibition", *J. Colloid Interf. Sci..*, 304, 442-458, 2006.

Ramiere I., Angot P., Belliard M., "A fictitious domain approach with spread interface for elliptic problems with general boundary conditions", *Comput. Methods Appl. Mech. Engrg*., 196, 4-6, 766-781, 2007.

Sterpi, D., "Effects of the erosion and transport of fine particles due to seepage flow", *international journal of Geomechanics*, vol 3, n°2, 111-122, 2003.

Sussman, M., Smereka, P., Osher, S., "A level set approach for computing solutions to incompressible two-phase flows", *J. Comput. Phys.*, 114:146-159, 1994.

Vardoulakis I, Stavropoulou M, Papanstasiou P. "Hydromechanical aspects of sand production problem", *Transport in Porous Med*, **22**, 225-244, 1996.

Vardoulakis, I., Papanastasiou, P., Stavropoulou, M., "Sand Erosion in Axial Flow Conditions", *Transport Porous Med*, Vol. 45, 267–281, 2001.

Vincent S., Caltagirone J.P., Lubin P., Randrianarivelo T.N., "An adaptative augmented Lagrangian method for three-dimensional multimaterial flows", *Comput. Fluids*, 33 (10), 1273-1289, 2004.