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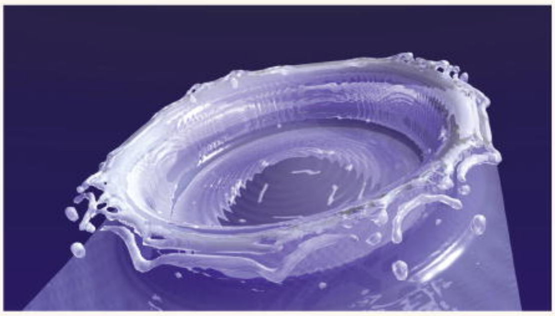

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One-dimensional modeling of piping flow erosion

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Abstract

A process called "piping", which often occurs in water-retaining structures (earth-dams, dykes, levees), involving the formation and progression of a continuous tunnel between the upstream and downstream sides, is one of the main cause of structure failure. Starting with the diphasic flow volume equations and the jump equations including the erosion processes, a simplified one-dimensional model for two-phase piping flow erosion was developed. The numerical simulation based on constant input and output pressures showed that the particle concentration can be a significant factor at the very beginning of the process, resulting in the enlargement of the hole at the exit. However, it was concluded that this influence is a secondary factor: the dilute flow assumption, which considerably simplifies the description, is relevant here. ***To cite this article: D. Lachouette et al., C. R. Mecanique 336 (2008).***

Résumé

Modélisation unidimensionnelle du renard hydraulique. Un phénomène appelé renard hydraulique concerne de nombreux ouvrages hydrauliques (barrages, digues, levées), conduisant à la formation et au développement d'un tunnel continu entre l'amont et l'aval. C'est l'une des causes principales de rupture. A partir des équations d'écoulement diphasique, et des équations de saut avec érosion, nous proposons un modèle simplifié unidimensionnel pour les écoulements denses de conduit avec érosion. Les résultats numériques avec pression constante en entrée et en sortie montrent que l'influence de la concentration en particules solides peut être significative au début de l'évolution, conduisant à un élargissement du trou en sortie. Toutefois, nous concluons que cette influence est du second ordre : l'hypothèse d'écoulement dilué, qui simplifie considérablement la description, est pertinente. ***Pour citer cet article : D. Lachouette et al., C. R. Mecanique 336 (2008).***

Keywords: Fluid mechanics; Two-phase flow; Internal erosion; Piping flow erosion

Mots-clés : Mécanique des fluides ; Écoulements diphasiques ; Érosion interne ; Renard hydraulique

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1. Introduction

Piping erosion is one of the main causes responsible for the failure of water retaining structures (such as dams, dykes and levees). This process involves the formation and progression of a continuous tunnel between the upstream and downstream sides. The piping erosion process involves pressure-driven flows. The term "piping" is actually used in the geomechanics literature to denote two processes, namely (i) backward piping erosion, which is driven by a normal outward flow (the exfiltrating seepage causes the fluidization of grains at an exit face), and (ii) piping flow erosion, which is driven by a tangential flow (the pipe flow causes the removal of soil particles at the fluid/soil interface).

Several models have been developed for surface sand erosion under axial and radial flow conditions in the framework of continuum mixture theory [1,2]. The process of erosion is usually assumed in these models to involve a smooth transition from solid-like to fluid-like behaviour. This transition is described in term of a three-phase model (comprising solid, fluid, and fluidized phases). The present study deals with the piping erosion process under axial flow conditions: a continuous piping tunnel enlarged by a tangential flow of water. The present approach differs from previous theories in that our description deals with a sharp fluid/soil interface [3].

A model for interpreting the hole erosion test with a constant pressure drop was developed by Bonelli et al. [4,5]. This model yielded a characteristic erosion time depending on the initial hydraulic gradient and the coefficient of erosion. It was established that the product of the coefficient of erosion and the flow velocity is a significant dimensionless number: when this number is small, the erosion kinetics are low and the solid particles concentration does not affect the flow. This was found to be the case with most of the test results available in the literature. Based on the theoretical and experimental evidence presented, it was established that the evolution of the pipe radius during a process of erosion with a constant pressure drop obeys a scaling exponential law. However, the issue as to how the change of scale (from the laboratory to the actual structure) may affect the assumption that a dilute flow is involved still remains to be addressed. The present study was intended to provide a step in this direction.

2. Two-phase flow equations with interface erosion

It was proposed here to study the surface erosion occurring at a fluid/soil interface undergoing a turbulent flow process running parallel to the interface. The soil is eroded by the flow, which then carries away the eroded particles. As long as the particles are small enough in comparison with the characteristic length of the flow, this two-phase flow can be said to be a continuum. We take Ω to denote the volume of the two-phase mixture and Γ to denote the fluid/soil interface. For the sake of simplification, sedimentation and deposition processes are neglected. The mass conservation equations for the water/particle mixture and the eroded particle phase as well as the balance equation of momentum of the mixture within Ω can be written as follows the an Eulerian framework [3]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad \frac{\partial \rho Y}{\partial t} + \nabla \cdot (\rho Y \mathbf{u}) = -\nabla \cdot \mathbf{j}, \quad \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = \nabla \cdot \mathbf{T} \tag{1}$$

In these equations, ρ is the density mixture, depending on the particle mass fraction Y, $\mathbf{u}$ is the mass-weighted average velocity, $\mathbf{j}$ is the mass diffusion flux of particles, and $\mathbf{T}$ is the Cauchy stress tensor in the mixture.

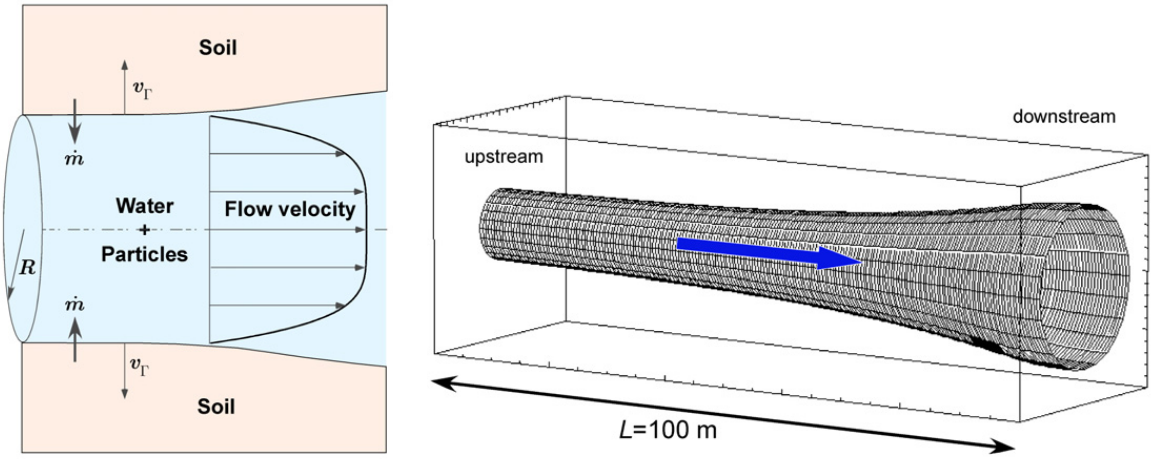
As there is a process of erosion, a mass flux crosses the interface Γ. Let us take $\mathbf{n}$ to denote the normal unit vector of Γ oriented outwards from the flow, and $\mathbf{v}_\Gamma$ to denote the normal velocity of Γ. The jump equations over Γ are [3]

$$[\![\rho(\mathbf{v}_\Gamma - \mathbf{u}) \cdot \mathbf{n}]\!] = 0, \quad [\![\rho Y(\mathbf{v}_\Gamma - \mathbf{u}) \cdot \mathbf{n}]\!] = [\![\mathbf{j} \cdot \mathbf{n}]\!], \quad [\![\rho \mathbf{u}(\mathbf{v}_\Gamma - \mathbf{u}) \cdot \mathbf{n}]\!] = -[\![\mathbf{T} \cdot \mathbf{n}]\!] \tag{2}$$

where $[\![a]\!] = a_b - a_{\text{soil}}$ is the jump of any physical variable a across the interface, and a_b and a_{soil} stands for the limiting value of a on the flow and soil sides of the interface, respectively. The soil is taken to be homogeneous, water saturated, rigid and devoid of seepage. The coordinate system depends on the soil in question ($\mathbf{u}_{\text{soil}} = 0$). The total flux of eroded material (consisting of both particles and water) crossing the interface is therefore $\dot{m} = \rho_{\text{soil}} \mathbf{v}_\Gamma \cdot \mathbf{n}$, where ρ_{soil} is the density of the saturated soil.

Erosion laws dealing with soil surface erosion by a tangential flow are often written as threshold laws such as:

$$\dot{m} = \begin{cases} k_{\text{er}}\big(|\tau_b| - \tau_c\big) & \text{if } |\tau_b| > \tau_c, \\ 0 & \text{otherwise} \end{cases} \tag{3}$$



(a) (b)

Fig. 1. Axisymmetrical flow with soil erosion and transport of the eroded particles.

where $|\tau| = \sqrt{(\mathbf{T}\cdot\mathbf{n})^2 - (\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n})^2}$ is the tangential shear stress at the interface, τ_c (Pa) is the critical shear stress and k_{er} (s/m) is the coefficient of soil erosion.

This complete set of Eqs. (1)–(3) was previously used to study various situations involving permanent boundary layer flow over an erodable soil [3], or for modeling dilute zero-dimensional piping flows with erosion by performing a spatial integration over Ω [4,5]. The use of these equations can be extended here to studies on one-dimensional piping flow with erosion.

3. Application to dense piping flows with erosion

Let us take a cylinder Ω with current radius $R(x,t)$ (initial value R_0), and length L (Fig. 1). The equations above can be simplified in a boundary layer theory spirit [6] in order to obtain the Reduced Navier Stokes/Prandtl equations [5,7]. The system obtained can therefore be integrated on a cross section. For the sake of simplification, the tangential velocities are assumed to be continuous across Γ, and the influence of the concentration on the velocity profile is neglected.

We take $\bar{u}(x,t)$ to denote the average longitudinal velocity, $p(x,t)$ to denote the pressure and $\bar{\phi}(x,t)$ to denote the average volume fraction of the solid phase in the mixture, where x denotes the axial coordinate and $\bar{a}$ denotes the mean value of any quantity a across a section. The following system is finally obtained:

$$\frac{\partial R}{\partial t} = \frac{\dot m}{\rho_{\mathrm{soil}}}\left(1+\left(\frac{\partial R}{\partial x}\right)^2\right)^{1/2} \quad \text{(total mass jump equation)} \tag{4}$$

$$\frac{\partial \bar u}{\partial x} + 2\frac{\bar u}{R}\frac{\partial R}{\partial x} = 0 \quad \text{(total mass balance equation)} \tag{5}$$

$$\frac{\partial \bar\phi}{\partial t} + \beta\bar u\frac{\partial \bar\phi}{\partial x} = \frac{2\dot m}{R\rho_{\mathrm{soil}}}(\phi_{\mathrm{soil}} - \bar\phi) \quad \text{(sediment mass balance equation)} \tag{6}$$

$$\bar\rho\left(\frac{\partial \bar u}{\partial t} + \beta\bar u\frac{\partial \bar u}{\partial x}\right) = \frac{2}{R}(\tau_b - \beta'\bar u\dot m) - \frac{\partial p}{\partial x} \quad \text{(axial momentum balance equation)} \tag{7}$$

where

$$\beta = \frac{\overline{u^2}}{\bar u^2}, \qquad \beta' = 1 + (\beta-1)\left(1-\frac{\bar\rho}{\rho_{\mathrm{soil}}}\right) \tag{8}$$

The variable mixture density is $\rho = \phi(\rho^p - \rho^w) + \rho^w$, where ρ^p and ρ^w are the (constant) soil particle and water densities, respectively. Volume fraction and mass fraction are related as follows: $\rho^p\phi = \rho Y$. The constant saturated soil density is $\rho_{\mathrm{soil}} = (\rho^p - \rho^w)\phi_{\mathrm{soil}} + \rho^w$ where $\phi_{\mathrm{soil}} = 1 - n$ is the compacity of the soil, while n is the porosity.

Table 1
Numerical values of the parameters

R_0	L	p^*_{in}	c_B	d^p	c_l	ρ^p	ρ^w	ϕ_{soil}	τ_c	k_{er}
3 mm	100 m	3.33 MPa	0.01	1 mm	0.1	2700 kg/m^3	1000 kg/m^3	0.5	10 Pa	10^{-2} s/m

Although some quite sophisticated turbulence models are now being commonly used to simulate single-phase fluid flows, the same cannot be said for the two-phase flows involving heavy particles of varying concentration. We use a strong assumption for the behaviour law of this flow:

$$\tau_b = -\bar{\rho} f_w(R_e) f_m(\bar{\phi})\bar{u}^2 \tag{9}$$

The effect of the concentration were accounted for using Julien's quadratic rheological law [8]. The effect of the Reynolds number was modeled as described by Barenblatt [9]:

$$f_m(\bar{\phi}) = 1 + c_B \frac{\rho^d}{\bar{\rho}}\left(\frac{d^d}{l_m}\right)^2 \lambda(\bar{\phi}), \qquad \lambda(\bar{\phi}) = \left[\left(\frac{\phi_{\text{soil}}}{\bar{\phi}}\right)^{1/3} - 1\right]^{-1} \tag{10}$$

$$f_w(R_e) = \left(\frac{2^{\alpha}\alpha_e(1+\alpha_e)(2+\alpha_e)}{\mathrm{e}^{3/2}(\sqrt{3}+5\alpha_e)}\right)^{2/(1+\alpha_e)}, \qquad \alpha_e = \frac{3}{2\ln R_e} \tag{11}$$

where c_B is the Bagnold coefficient, d^p is the (eroded) soil particle diameter, l_m is the mixing length, and R_e is the Reynolds number. The mixing length is taken to be $l_m = c_l R$ where $c_l < 1$ is a constant. The coefficient β is evaluated with the radial velocity profile proposed by Barenblatt [9] for modeling turbulent pipe flows:

$$\beta = \frac{(1+\alpha_e)(2+\alpha_e)^2}{4(1+2\alpha_e)} \tag{12}$$

4. Numerical results

Eqs. (4)–(12) were solved numerically with the unknowns $(\bar{\phi}, R, p, \bar{u})$, and the following conditions:

$$\text{Initial conditions:} \quad R = R_0, \quad \bar{\phi} = 0, \quad \bar{u} = \bar{u}_0, \quad p = \frac{p^*_{\text{in}}(L-x)}{L}, \quad t = 0,\ 0 \leqslant x \leqslant L \tag{13}$$

$$\text{Boundary conditions:} \quad \bar{\phi}(t,0) = 0, \quad p(t,0) = p^*_{\text{in}}, \quad p(t,L) = 0 \tag{14}$$

The boundary conditions required a specific numerical procedure, which is not described here in detail for the sake of conciseness. The initial condition corresponds to the stationary solution of the pipe flow without any erosion. Erosion occurs if $P_0 > \tau_c$, where $P_0 = R_0 p^*_{\text{in}}/(2L)$ is the initial driving pressure. Numerical values of the parameters are given in Table 1. These values correspond to a pipe located in a large dam with a highly erodable soil [5]. This situation involves a long pipe (100 m) and a very high head drop (300 m). Under these conditions, the accumulated eroded particles are liable to transform the flow into a concentrated suspension. However, this situation is more extreme than any of the most critical situations encountered so far in reality.

Fig. 2 gives the dimensionless quantities as a function of the dimensionless time t/t_{er}: the radius R/R_0, the pressure p/p^*_{in}, the volume particle concentration $\bar{\phi}/\phi_{\text{soil}}$, the tangential stress τ_b/τ_{b0} (where τ_{b0} is the initial tangential stress), and the velocity $\bar{u}/\bar{u}_0$. The characteristic erosion time is $t_{\text{er}} = 2L\rho_{\text{soil}}/(k_{\text{er}} p^*_{\text{in}})$ [4,5], and its value is $t_{\text{er}} = 11.11$ s.

In order to determine the effect of the concentration, four stages were identified:

- $t/t_{\text{er}} = 0$–2.2; in the downstream part, the accumulated eroded particles transform the flow into a concentrated suspension ($\bar{\phi}/\phi_{\text{soil}} = 1$), causing high tangential stress values; this results in an enlargement of the pipe in the downstream part and low velocity values;
- $t/t_{\text{er}} = 2.2$–4.5; the concentration decreases but remains high;
- $t/t_{\text{er}} = 4.5$–6.8; the flow becomes dilute ($\bar{\phi}/\phi \to 10\%$);
- $t/t_{\text{er}} > 6.8$–15.7; the flow is dilute ($\bar{\phi}/\phi < 10\%$); the behaviour of the piping erosion flow is not affected by the concentration, which becomes a secondary variable.

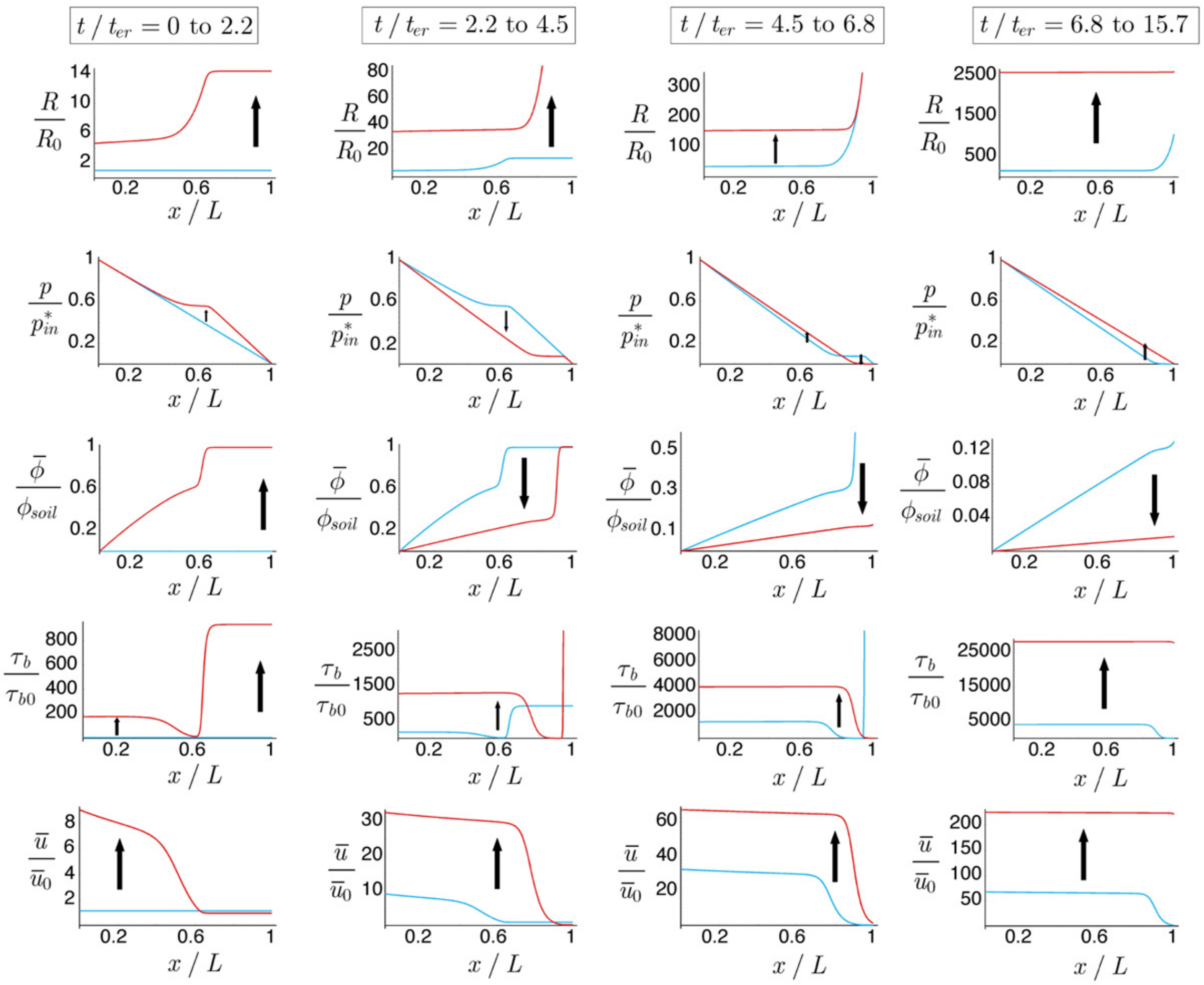


Fig. 2. Changes with time in the dimensionless radius R/R_0, pressure p/p^*_{in}, concentration $\bar{\phi}/\phi_{\text{soil}}$, tangential stress τ_b/τ_{b0} and velocity $\bar{u}/\bar{u}_0$.

Fig. 1(b) gives the three-dimensional plot of the hole during the first stage (not to scale). Taking into account the many simplifying assumptions about the radial concentration and velocity profiles, this overall picture shows that this simple model accurately simulate some of the main features of piping flow erosion.

5. Conclusion

It has been established in previous studies that the dilute flow assumption is relevant when modeling piping erosion flow under laboratory conditions. To address the issue as how the change of scale (from the laboratory to the actual structure) might affect this assumption, a simplified one-dimensional model for piping erosion two-phase flows was used to simulate some of the main features of the process. At this stage, the model was not expected to yield highly accurate results: we focused explicitly here on the more limited goal of obtaining some quantitative data and orders of magnitude. Numerical simulations carried out under constant input and output pressure conditions showed that the particle concentration can have significant effects at the very beginning of the process, resulting in the enlargement of the hole at the exit. However, this factor was found to be subsequently negligible. The dilute flow assumption, which considerably simplifies the description, is therefore relevant for modeling piping erosion flows.

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